UNIT I
ELECTROSTATICS

WEIGHTAGE: 8 marks

- **Properties of charges**
- **Coulomb's law of electrostatics**
- **Principle of superposition**
- **Electric field due to a point charge**
- **Electric field lines**
- **Electric dipole- electric field at axial and equatorial line, torque, potential energy**
- **Electric flux**
- **Gauss's law and its applications**
- **Electric potential- point charge and electric dipole**
- **Equipotential surface and its properties**
- **Conductors and insulators**
- **Capacitance- parallel plate capacitor**
- **Combination of capacitors**
- **Energy stored by capacitor**

GIST OF THE LESSON

1. **Electrostatics:** The branch of Physics which deals with charges at rest, the force between the charges, field and potential between the charges.
2. **Charge:** It is something possessed by material objects that makes it possible for them to exert electrical force and to respond to electrical force.
3. **Properties of charges:**
   (a) **Quantisation of charge**: It is property by virtue of which all free charges are integral multiple of a basic unit of charge of an electron.

   $$\boldsymbol{q = \pm ne} \text{ where } e=1.6\text{x}10^{-19}$$

   (b) **Additive nature of charge**: It is property by virtue of which total charge of a system is obtained by simply adding algebraically all charges present any where on the system.

   $$q = q_1 + q_2 + q_{3------} + q_n$$

   (c) **Conservation of charge**: It is property by virtue of which total charge of an isolated system always remains constant.
4. **Coulombs law:** The force of interaction between two point charges is directly proportional to the product of charges and inversely proportional to square of distance between them.

   $$\boldsymbol{F \propto q_1 q_2} \quad \text{and} \quad \boldsymbol{F \propto \frac{1}{r^2}}$$

   $$\boldsymbol{F = k\frac{q_1 q_2}{r^2}}$$

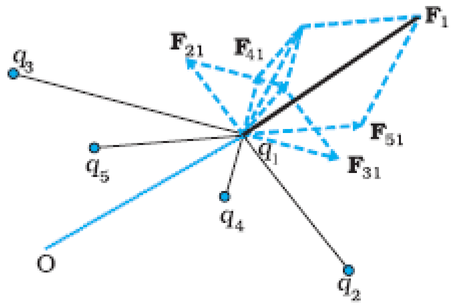
   Where k is a constant which depends on system of measurement and nature of medium.

   $$\boldsymbol{k = \frac{1}{4\pi\epsilon_0}} = 9 \times 10^9 \text{Nm}^2/\text{kg}^2$$
5. **Unit of charge: SI unit** of charge is one coulomb which is that charge which when placed at a distance of 1m from an equal charge and similar charge in vacuum would repel it by a force of $9\text{x}10^9$newtons.

<u>CGS</u> unit is 1 statcoulomb or 1 electrostatic unit

1coulomb = 3x10[9] stat coulomb

6. **Force between multiple charges:** According to principle of superposition, total force on any charge due to a number of charges at rest is vector sum of all the forces on that charge due to other charges, taken one at a time.



$$F_1 = F_{12} + F_{13} + F_{14} \dots\dots\dots.. + F_{1n}$$

$$F_1 = k\sum_{i=1}^{n}\frac{q_1 q_i}{r_{1i}^2}$$

7. **Electric field:** Due to a given charge is the place space around a given charge in which force of attraction or repulsion due to the charge can be experienced by any other charge.
8. **Electric field intensity:** At any point is the strength of field at that point. It is defined as the force experienced by unit positive charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0}$$

9. **Electric field intensity due to a point charge:**

$$\vec{E} = k\frac{q}{r^2}$$

10. **Unit of electric field intensity:** The SI unit of electric field is **newton per coulomb.**
11. **Electric field intensity due to multiple charges:**
    Electric field intensity at a point due to a group of charges is equal to the vector sum of the electric field intensity due to individual charges at the same point.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \dots\dots\dots + \vec{E}_N$$

$$\vec{E} = k\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

12. **Electric field lines:** It is the path straight or curved in electric field, such that tangent at any point of it gives direction of electric field at that point.
    Properties of electric field lines:
    1. Electric field lines are discontinuous curves. They start from positive charge and end at negative charge.
    2. Tangent to electric field line at any point gives direction of electric field at that point.
    3. No two lines of force can intersect each other because at the point of intersection , there will be two possible direction of electric field which is not possible. Hence the lines do not cross each other.
    4. The electric field lines are always normal to the surface of conductor.
    5. The electric field lines contract longitudinally, on account of attraction between unlike charges.

6. The electric field lines exert a lateral pressure on account of repulsion between like charges.

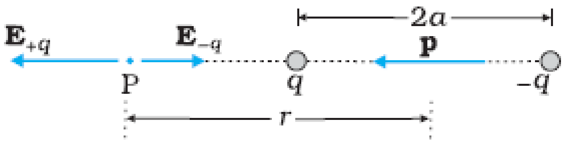
13. **<u>Electric dipole:</u>** It is a system of equal and opposite charges separated by a small distance.

14. **<u>Dipole moment:</u>** It is given by product of magnitude of either charge and distance between the two charges.

$$\vec{p} = q(2\vec{a})$$

The **direction** of dipole moment is from is from ***positive to negative*** charge

15. **<u>Field intensity on axial line of dipole;</u>**



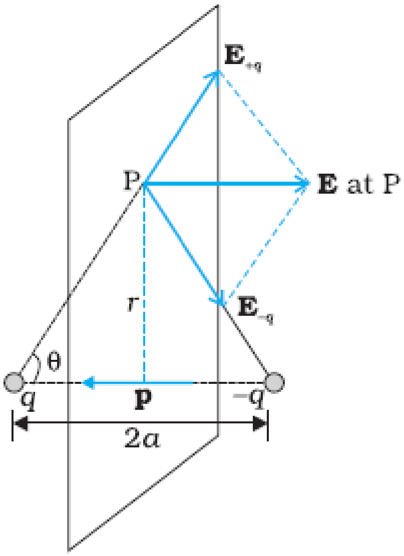
$$E = k\frac{2pr}{(r^2 - a^2)}$$

When a << r, $E = \frac{2p}{r^3}$

16. **<u>Field intensity at a point on the equatorial line of dipole:</u>**

$$\boldsymbol{E = k\frac{p}{(r^2 + a^2)^{3/2}}}$$

$$\boldsymbol{When\ a \ll r, E = k\frac{p}{r^3}}$$



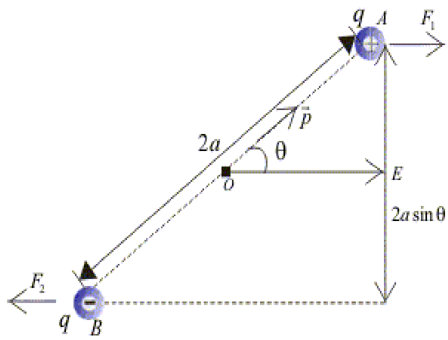
17. **<u>Torque on a dipole in uniform electric field:</u>**

Torque on dipole in uniform electric field

$$\boldsymbol{\tau = pE\sin\theta}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$$

Torque is perpendicular to both dipole moment vector and electric field vector



18. **<u>Electric flux:</u>** It is represented by electric field passing normally through a given surface. SI unit of flux is **newton m^2/coulomb.** It is a **scalar quantity.**

$$\Delta\emptyset = \vec{E}.\overrightarrow{\Delta S} = E\Delta S\cos\theta$$

19. **Gauss's Law:** 'Electric flux over a closed surface is $1/\varepsilon_0$ times the charge enclosed by it.'

$$\emptyset = \frac{q}{\epsilon_0}$$

20. **Electric field due to a an infinite long uniformly charged wire:** For a line charge with uniform charge density λ, electric field at a distance r is

$$\boldsymbol{E} = \boldsymbol{k}\frac{\boldsymbol{2\lambda}}{\boldsymbol{r}}$$

21. **Electric field due to a uniformly charged spherical shell:** Let R be the radius of uniformly charged shell with charge density $'\sigma'$. Electric field at a distance r when

r › R $\quad E = k\frac{q}{r^2}$

r=R $\quad E = k\frac{q}{R^2} = \frac{\sigma}{\epsilon_0}$

r‹ R $\quad$ E=0

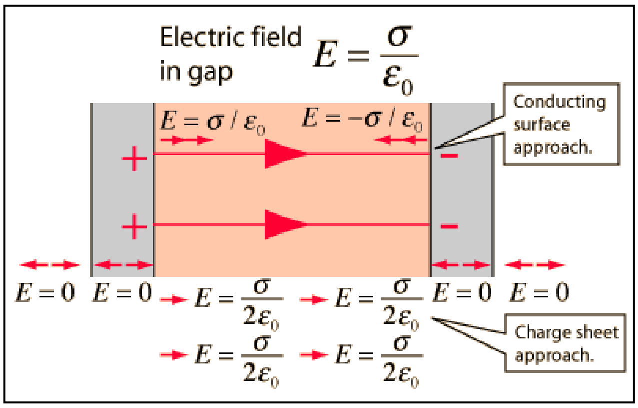
22. **Electric field due to a thin infinite plane sheet of charge:** Let σ be the surface charge density on the sheet. E.F is independent of the distance from the plane sheet.

$$E = \frac{\sigma}{2\epsilon_0}$$

23. **Electric field due to two thin parallel sheet of charge:** Electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0}$$

and in the region on either side of the plates $E = 0$



24. **Electrostatic potential difference**: P.D between two points in electric field is defined as the amount of work done to move a test charge without acceleration from one point to another. **SI unit** of PD is **volt.** $\quad \Delta \boldsymbol{V} = \frac{\boldsymbol{W_{AB}}}{\boldsymbol{q}}$

25. **Electrostatic potential:** Electrostatic potential at any point in electric field is the amount of work done in moving a unit positive charge from infinity to the point.

$$\boldsymbol{V} = \frac{\boldsymbol{W_{\infty B}}}{\boldsymbol{q}} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{r}}$$

Potential is a **scalar quantity** measured in **volts.**

26. **Electrostatic potential at any point due to a dipole:** Potential at a distance r from the centre of dipole at an angle θ with the axis of dipole is

$$V = k\frac{p\cos\theta}{r^2 - a^2}$$

At a point on the axis of dipole θ=0

$$V = k\frac{p}{r^2 - a^2}$$

At a point on the equatorial line of dipole θ=90

$$V = 0 \text{ as } \cos 90 = 0$$

27. **Equipotential surface**: It is the surface at every point of which the potential is same.

Properties of equipotential surface:

1. No work is done in moving a charge from one point of equipotential surface to the other
2. For any charge configuration, equipotential surface through a point is normal to the electric field at that point.
3. Where electric field is large the distance between electric field is small and vice versa.

28. **Potential energy of system of charges:** It is defined as the amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.

29. **Expression for potential energy for a system of charges:**

$$\boldsymbol{U} = \frac{1}{2}k\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\frac{q_i q_j}{r_{ij}}$$

30. **Electrostatics of a conductors:**

1. Electric field inside a conductor is zero
2. The interior of a conductor can have no excess charge in static situations.
3. Electric field just outside the conductor is normal to the surface of the conductor.
4. Electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface.
5. Electric field at the surface of conductor is $E = \frac{\sigma}{\epsilon_0}$

31. **Relation between electric potential and electric intensity:**

$$E = -\frac{dV}{dr}$$

32. **Electrical capacitance**: It is ability to store charge. It is numerically the charge required to raise the potential by unity.

$$C = \frac{Q}{V}$$

SI unit of capacity is Farad

$$1 Farad = \frac{1 Coulomb}{1\ Volt}$$

$$C = [M^{-1}L^{-2}T^{4}A^{2}]$$

33. **Capacity of isolated spherical conductor:** Let R be the radius of spherical conductor.

$$C = 4\pi \epsilon_0 R$$

34. **Capacity of a parallel plate capacitor:** Let the area of plates be **A,** distance between the plates be **d,** surface charge density be **σ** and **air** is medium between the plates

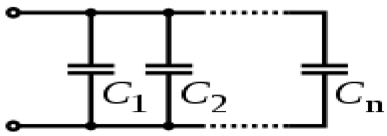
$$C_0 = \frac{\epsilon_0 A}{d}$$

35. **Capacity of a parallel plate capacitor with dielectric**: If k is dielectric constant of the medium between the plates

$$C_m = \frac{k \in_0 A}{d}$$

The dielectric constant of the medium is given by

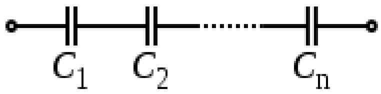
$$k = \frac{C_m}{C_0}$$



36. <u>Grouping of capacitors:</u>

<u>Parallel combination:</u>

$$\boldsymbol{C = C_1 + C_2 \pm - - - - - - - + C_n}$$



<u>Series combination:</u>

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots \ldots . . + \frac{1}{C_n}$$

37. <u>Energy stored in capacitor:</u>

Energy is stored in the dielectric medium between the plates of capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\text{QV}$$

When a dielectric is inserted between the plates of capacitor and the battery remains connected

$$U = \frac{1}{2}(kC)V^2 = U = k\frac{1}{2}CV^2 = kU_0$$

Total energy is additive in series and parallel combination.

$$U = U_1 + U_2 + U_3$$

<u>FORMULAS</u>

<u>IMPORTANT FORMULAE IN ELECTROSTATICS</u>

1. Electrostatic force between two charges

   $F = K.\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0\epsilon_r}.\frac{q_1 q_2}{r^2}$

   For air, $\epsilon_r = 1$

   Fair = $\frac{1}{4\pi\epsilon_0}.\frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{q_1 q_2}{r^2}$

2. Electric field intensity due to a point charge, $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_o}$
3. Electric field intensity due to infinite linear charge density (λ)

   $E = \frac{1}{4\pi\epsilon_0}.\frac{2\lambda}{r}$

4. Electric field intensity near an infinite thin sheet of surface charge density σ

   $E = \frac{\sigma}{2\epsilon_0}$

   For thick sheet $= \frac{\sigma}{\epsilon_0}$.

5. Electric potential, $V = \lim_{q_0 \to 0} \frac{w}{q_o}$

   Electric potential due to a point charge, $V = \frac{1}{4\pi\epsilon_0}.\frac{q}{r}$

6. Relation between electric field and potential $E = -\frac{dV}{dr} = \frac{V}{r}$ (numerically)
7. Dipole moment, $\vec{P} = q.2\vec{l}$
8. Torque on a dipole in uniform electric field, $\vec{\tau} = \vec{p} \times \vec{E}$.
9. Potential energy of dipole, U$= -\vec{p}.\vec{E} = -pE\cos\theta$
10. Work done in rotating the dipole in uniform electric field from orientation Q_1 to Q_2 is

    $W = U_2 - U_1 = pE(\cos\theta_1 - \cos\theta_2)$
11. Electric field due to a short dipole
    (i) at axial point, $E_{axis} = \frac{1}{4\pi\epsilon_0}.\frac{2p}{r^3}$
    (ii) at equatorial point, $E_1 = \frac{1}{4\pi\epsilon_0}.\frac{p}{r^3}$
12. Electric potential due to a short dipole
    (i) At axial point, $V_{axis} = \frac{1}{4\pi\epsilon_0}.\frac{p}{r^2}$
    (ii) At equatorial point, $V = 0$.
13. Dielectric constant, $K = \frac{\epsilon}{\epsilon_0} = \frac{C_{med}}{C_{air}}$
14. Capacitance of parallel plate capacitor
    (i) $C = \frac{A\epsilon_0 K}{d}$, in medium of dielectric constant K
    (ii) $C = \frac{A\epsilon_0}{d-t(1-\frac{1}{K})}$; if space between plate partially filled with dielectric of thickness t.
15. Combination of capacitors :-
    (i) In series, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$, $q_1 = q_2 = q_3$, $V = V_1 + V_2 + V_3$
    (ii) In parallel, C$= C_1 + C_2 + C_3$, $q = q_1 + q_2 + q_3$, $V_1 = V_2 = V_3 = V$

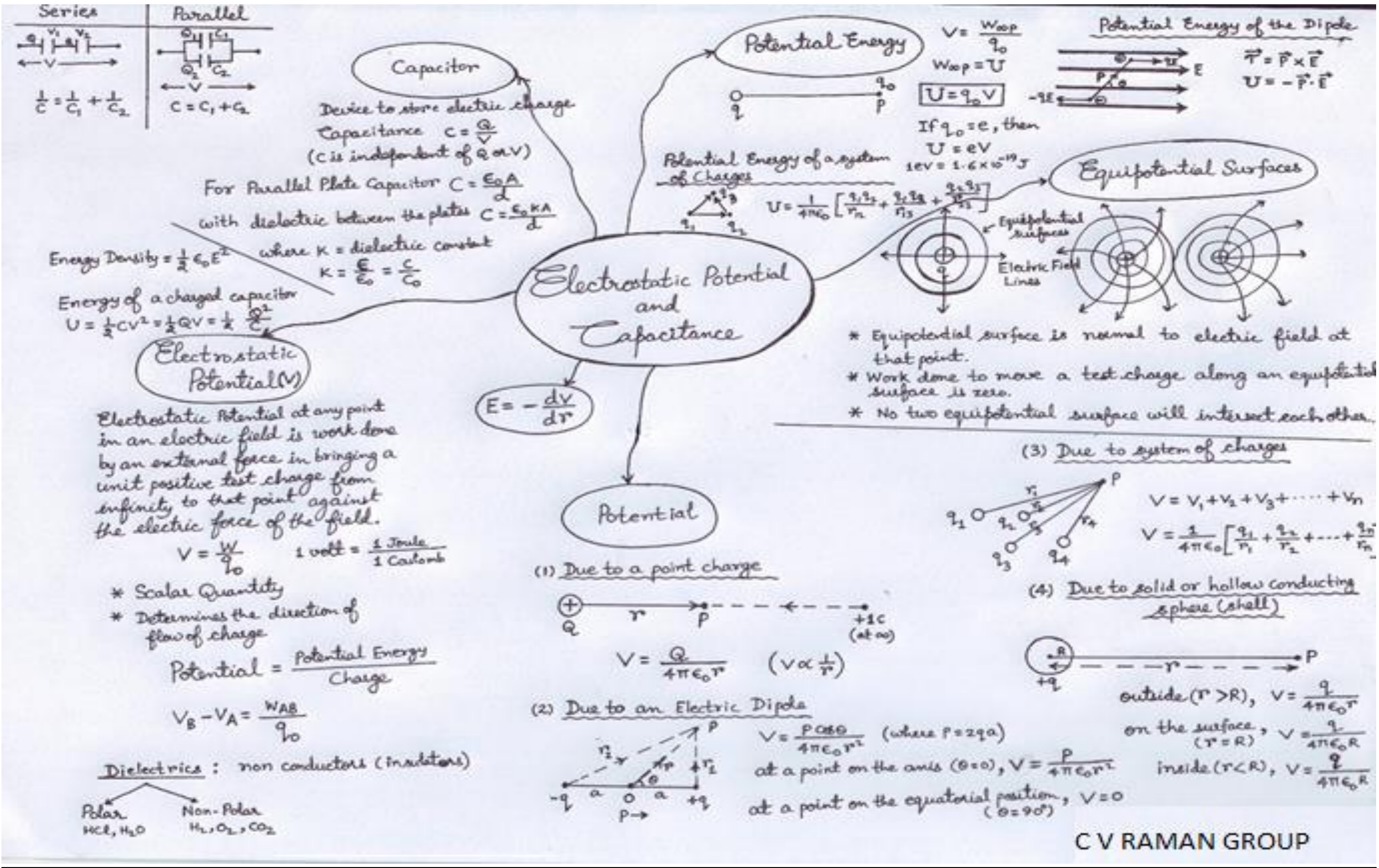
16. Energy stored by capacitor

    U$= \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$
17. Electrostatic energy density

    $\vartheta_e = \frac{1}{2}\epsilon_0 E^2$, in air

    $\vartheta_e = \frac{1}{2}\epsilon E^2$, in medium
18. Total electric flux, $\Phi = \oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} \times net\ charge\ enclosed\ by\ the\ surface$

MIND MAP

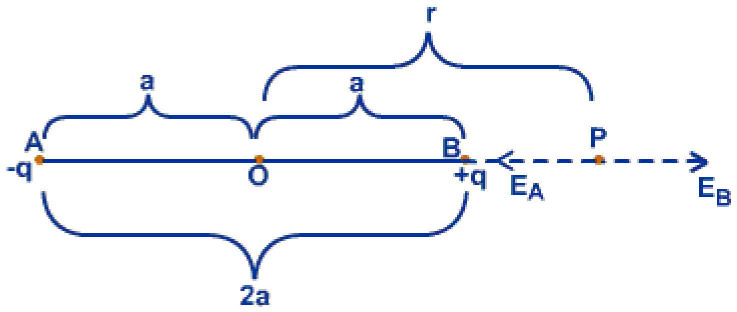


IMPORTANT TOPICS (MLL)

- **Statement of Guass Theorem and its application.**
- **Electric field due to infinite plane sheet of charge**
- **Electric field due to spherical shell**
- **Electric field due to infinite uniformly charged line charge**
- **Electric dipole- torque acting on the dipole,electric field on axial and equatorial line.**
- **Enegy stored in a capacitor.**
- **Capacity of a parallel plate capacitor with (i) air (ii) dielectric (iii) conducting medium between the plates**
- **Electric potential due to dipole and point charge.**
- **Numericals on series and parallel combination of capacitoer.**
- **Coulombs law**
- **Electrostatic Potential energy and equipotential surfaces**

3 MARKS & 5 MARKS QUESTIONS

1. Derive expression for electric field at a point on the axial line of the dipole. Githe direction of electric field at the point.
   The axial line of a dipole is the line passing through the positive and negative charges of the electric dipole.



Consider a system of charges (-q and +q) separated by a distance 2a. Let 'P' be any point on an axis where the field intensity is to be determined.

Electric field at P (E_B) due to +q

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(BP)^2} \quad \text{along BP}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

Electric field at P due to -q (E_A)

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(AP)^2} \quad \text{along PA}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{(r+a)^2}$$

Net field at P is given by

$$E_p = E_B - E_A$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}\right]$$

Simplifying, we get

$$E_p = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{\left(r^2-a^2\right)^2}$$

$$E_p = \frac{2qa}{4\pi\varepsilon_0}\frac{2r}{\left(r^2-a^2\right)^2}$$

$$\left(2aq = p\ ,\ K = \frac{1}{4\pi\varepsilon_0}\right)$$

or $E_p = \dfrac{2kpr}{\left(r^2 - a^2\right)^2}$ along BP

As a special case :

If $2a \ll r$, $E_p = \dfrac{2kp}{r^3}$ along BP

2. Derive expression for electric field at a point on the equatorial line of dipole.
An equatorial line of a dipole is the line perpendicular to the axial line and passing through a point mid way between the charges.

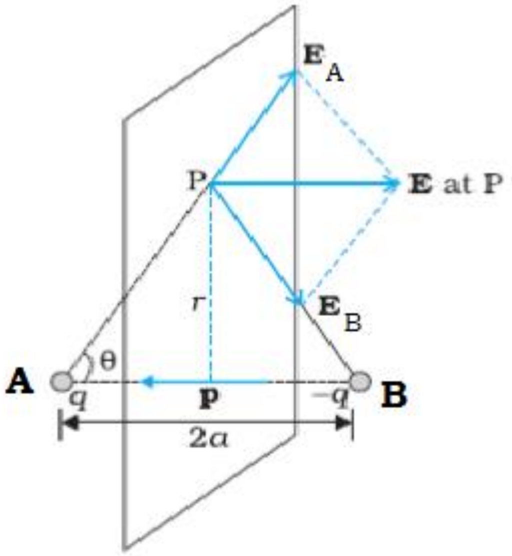
Consider a dipole consisting of -q and +q separated by a distance 2a. Let P be a point Consider a point P on the equatorial line.

$\vec{E}_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(AP)^2}$ along PA

$\vec{E}_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(r^2 + a^2)}$

$\vec{E}_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(BP)^2}$ along BP

$\vec{E}_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(r^2 + a^2)}$



The resultant intensity is the vector sum of the intensities along PA and PB. E_A and E_B can be resolved into vertical and horizontal components. The vertical components of E_A Sinθ and E_B Sinθ cancel each other as they are equal and oppositely directed. It is the horizontal components which add up to give the resultant field.

$E = E_A \cos\theta + E_B \cos\theta$

$E_A = E_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\left(\sqrt{r^2 + a^2}\right)^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\left(r^2 + a^2\right)}$

$E = 2E_A \cos\theta$

Substituting, $\cos\theta = \dfrac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}$ in the above equation

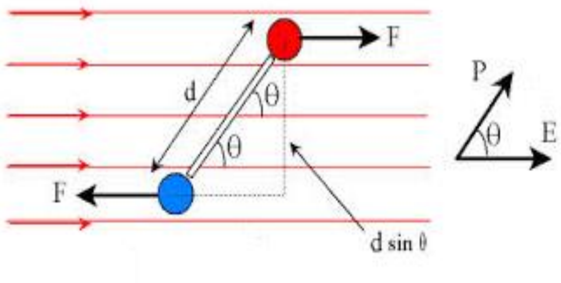
$E = 2E_A \cos\theta = \dfrac{2}{4\pi\varepsilon_0}\dfrac{q}{\left(r^2 + a^2\right)}\dfrac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}$

$$E = \frac{kp}{\left(r^2 + a^2\right)^{\frac{3}{2}}} \quad \text{along } P_x$$

As $2qa = p$

As a special case,

If $2a \ll r$ then, $E = \frac{kp}{r^3}$ along P_x



3. An electric dipole is held in uniform electric field
   (i) Show that no translator force acts on it.
   (ii) Derive an expression for he torque acting on it

Force on +q charge=qE along direction of E

Force on –q charge =qE opposite to E

F_{net}=qE-qE =0

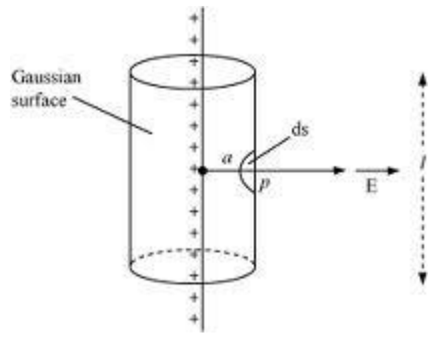
The forces are equal in magnitude, opposite in direction acting at different points, therefore they form a couple which rotates the dipole.

Torque$\tau = F \times perp.distance$

$\tau = F \times dsin\theta = qE \times dsin\theta = (qd)Esin\theta$

$[\tau = pEsin\theta \ Or \ \vec{\tau} = \vec{p} \times \vec{E}]$

4. State Gauss Theorem. A thin charged wire of infinite length has line charge density 'λ'. Derive expression for electric field at a distance 'r'.



Gauss's Law: 'Electric flux over a closed surface is $1/\varepsilon_0$ times the charge enclosed by it.'

$$\emptyset = \frac{q}{\epsilon_0}$$

To calculate the field at P we consider a Gaussian surface with wire as axis, radius r and length l as shown in the figure.
The electric lines of force are parallel to the end faces of the cylinder and hence the component of the field along the normal to the end faces is zero.
The field is radial everywhere and hence the electric flux crosses only through the curved surface of the cylinder.
If E is the electric field intensity at P, then the electric flux through the Gaussian surface is

$\emptyset = E \times 2\pi rl$

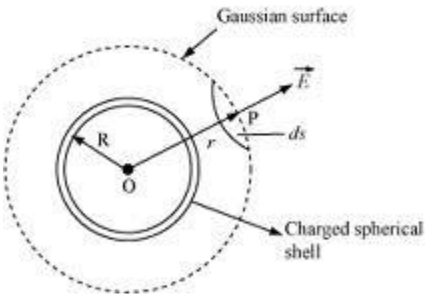
According to gauss theorem electric flux is

$$\emptyset = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

Hence$E \times 2\pi rl = \frac{\lambda l}{\epsilon_0}$

$$\left[\therefore \boldsymbol{E} = \frac{\boldsymbol{\lambda}}{\boldsymbol{2\pi \epsilon_0 r}}\right]$$

5. Charge q is distributed uniformly on a spherical shell of radius R. Using gauss law derive expression of electric field at a distance r from the centre when (i)r>R (ii) r=R (iii) r<R



Consider a hollow conducting sphere of radius R with its centre at O. let σ be its surface density. The field at any point P, outside or inside depends upon the distance from the centre of the spherical shell. Let the distance between the centre of the spherical shell and the point be r.

Case (i) r>R

At points outside the sphere the electric field is radial every where because of spherical symmetry.

Total electric flux$\emptyset = E \times 4\pi r^2$

According to gauss theorem electric flux is

$$\emptyset = \frac{q}{\epsilon_0}$$

hence$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$

$\left[E = \frac{q}{4\pi\epsilon_0 r^2}\right]$ Electric field due to charged shell is same as that due to a point charge q placed at the centre of shell

Case (i) r=R

When point P lies on the surface of the shell or sphere, r = R

hence$E \times 4\pi R^2 = \frac{q}{\epsilon_0}$

$$\left[E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{\sigma}{\epsilon_0}\right]$$

Case (i) r<R

The gaussian surface does not enclose any charge, (charge resides on the surface of the shell)

$E.4\pi R^2 = \frac{0}{\varepsilon_0}$ Hence $E = 0$

6. A parallel plate capacitor is charged by a battery to a potential V. It is disconnected and a dielectric slab is inserted to completely fill the space between the plates. How will
   (a) its capacitance

(b) electric field between the plates and

(c) energy stored in the capacitor be affected? Justify your answer in each case.

(a) On inserting dielectric capacity of the capacitor increases

$C_d = kC_a$

(b) Electric field decreases

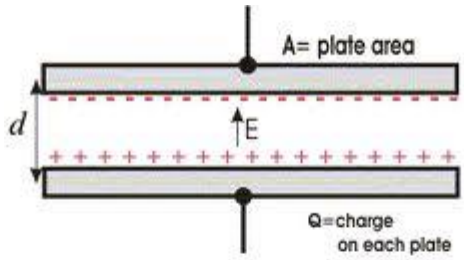
$E_d = \frac{E_a}{k}$

(c) As the battery is disconnected charge on the capacitor remains constant. Hence the energy stored in the capacitor is given by

$U = \frac{Q^2}{2C}$ As C increases therefore U decreases

$U_d = \frac{U_a}{k}$

7. Derive expression for capacity of parallel plate capacitor.



Let the surface charge density on the plates be σ

Such that $\sigma = \frac{Q}{A}$

Electric field between the plates is given by

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

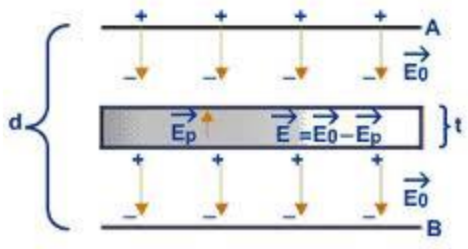
Potential difference between the plates is V=Ed

$$V = \frac{\sigma}{\epsilon_0} d$$

Capacity of a capacitor $C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\epsilon_0} = \frac{\epsilon_0 A}{d}$

$$\left[C = \frac{\epsilon_0 A}{d}\right]$$

8. Derive expression for capacity of parallel plate capacitor with dielectric as medium between the plates.



Let the surface charge density on the plates be σ

Such that $\sigma = \frac{Q}{A}$

Electric field between the plates is given by

$\overrightarrow{E_O} = \frac{\sigma}{\epsilon_0}$ and $\overrightarrow{E_\iota} = \frac{\sigma}{k\epsilon_0}$

where E_0 is electric field in air and E_i is electric field in dielectric.

Potential difference between the plates is given by

$$V = \overrightarrow{E_O}(d-t) + \overrightarrow{E_\iota}\, t = \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{k\,\epsilon_0} t = \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{k}\right)$$

Capacity of a capacitor $C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0}\left(d-t+\frac{t}{k}\right)} = \frac{\epsilon_0 A}{\left(d-t+\frac{t}{k}\right)}$

$$\left[C = \frac{\epsilon_0 A}{d - t(1-\frac{1}{k})}\right]$$

If d= t then $\left[C = k\frac{\epsilon_0 A}{d}\right]$

9. Derive expression for energy stored in a capacitor and hence obtain the expression for energy density of a parallel plate capacitor.

Consider a parallel plate capacitor of capacity C. Let at any instant the charge on the capacitor be Q'. Then potential difference between the plates will be

Suppose the charge on the plates increases by d Q'. The work done will be

$$dW = V'dQ' = \frac{Q'}{C}\,dQ'$$

The total work done is $W = \int_0^Q \frac{Q'}{C}\,dQ' = \left[\frac{Q^2}{2C}\right]$

This work done is stored as electrical potential energy.

$$\left[\therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}CV\right]$$

Energy density of parallel plate capacitor:

Capacity of a parallel plate capacitor is

$$C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2 = \frac{1}{2}\frac{\epsilon_0 A d^2 E^2}{d} = \frac{1}{2}\epsilon_0\,(Ad)E^2$$

$$\left[\boldsymbol{u = \frac{U}{Ad} = \frac{1}{2}\epsilon_0\,E^2}\right]$$

PREVIOUS YEAR QUESTIONS (1 marks)

1. Define dipole moment of an electric dipole. Is it a scalar or a vector?

Electric dipole moment of an electric dipole is equal to the product of either charge and distance between the two charges. $p = q \times 2a$ where p is dipole moment.
It is a scalar quantity.

2. Why must electrostatic field be normal to the surface at every point of a charged conduction?

The component of electric field along the tangent to the surface of the conductor must be zero.

$E\cos\theta = O$ where θ is angle between and tangent to the surface.
$E \neq 0, \therefore \cos\theta = 0\ Oor\ \theta = 90$
hence E is perpendicular to the surface

3. A proton is placed in a uniform electric field directed along the position x-axis. In which direction will it tend to move?

Proton will tend to move along X axis in the direction of electric field.

4. Why do the electric field lines not form closed loop?

No electric field exist from negative to p[ositive charge , hence electric field lines do not form closed loop.

5. In which orientation a dipole placed in a uniform electric field is in a) Stable, b) Unstable Equilibrium?
   (a) For stable equilibrium the angle between p and E must be 0^0
   (b) For unstable equilibrium the angle between p and E must be 180^0
6. Two point charges having equal charge are separated by 1m distance experience a force of 8N. What will be the force if they are held in water at the same distance? (Given $k_{water} = 80$)

$$\mathbf{k = \frac{F_a}{F_m} \quad \therefore\ F_m = \frac{F_a}{k} = \frac{8}{80} = \frac{1}{10}}$$

8. Point out right or wrong for the following statement
   a) The mutual force between two charges is not affected by the presence of other charges
   b) The potential, due to a dipole, at any point on its axial line, is zero.

a) Right- According to principle of superposition, force between two charges is not affected by presence of other charges.
b) Wrong- Potential due to a dipole is zero at equatorial line and not on axial line

9. A dipole, of dipole movement p is present in a uniform electric field E. Write the value of angle between p and E for which the torque experienced by the dipole is minimum.

$\tau = pE\sin\theta$ for the torque to be minimum $pE\sin\theta = 0\ \therefore\ \sin\theta = O\ or\ \theta = 0$

10. What is the electric potential due to electric dipole at an equatorial point?

Potential at a point on equatorial line is 0.

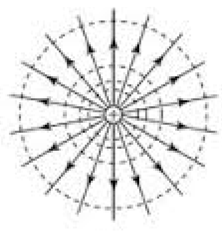
11. Two charges -2Q and +Q are located at points (a,0) and (4a,0) respectively. What is the flux through a sphere of radius '3a' with its centre at origin?

The total charge enclosed by sphere = -2Q +Q = -Q
According to Gauss theorem $\emptyset = \frac{q}{\epsilon_0}$

$$\emptyset = \frac{Q}{\epsilon_0}$$

12. What is the shape of equipotential surface due to a single isolated charge?

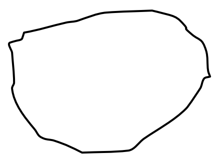
For an isolated charge equipotential surface are concentric spherical shells and distance between them increases with the decrease in field.

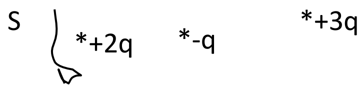


13. A charge q is placed at the centre of a cube of side l. What is the flux passing through each face of the cube?

According to gauss theorem electric flux linked with a closed surface is $\emptyset = \frac{q}{\epsilon_0}$
The flux is symmetrically distributed through all the six faces $\therefore\ \emptyset = \frac{1}{6}\frac{q}{\epsilon_0}$

14. Figure shows three pouint charges +2q, -q and +3q. What is the flux through the closed surface S?





Electric flux through the surface S

$$\emptyset = \frac{\sum q}{\epsilon_0} = \frac{+2q-q}{\epsilon_0} = \frac{q}{\epsilon_0}$$

15. If the radius of Gaussian surface is halved, how will the flux through the Gaussian surface change?

Even if the radius of the surface is halved, the charge enclosed by the surface does not change hence the flux remains constant.

16. A hollow metal sphere of radius 5 cms is charged such that potential on its surface is 5 V. What is the potential at the centre of the sphere?

In side a hollow sphere potential is constant and same as that on its surface.

Hence $V_i = V_S = 5\ V$

17. Name a physical quantity whose SI unit is J/C. Is it a scalar or a vector quantity?

J/C is unit of electric potential. It is a scalar quantity.

18. A point charge Q is placed at a point O as shown in the figure, Is the potential difference V_A-V_B positive, negative or zero, if Q is (i) positive (ii) negative.

Q*-----------------*A------------*B

$V_A - V_B = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$ **where $r_A › r_B$**

(i) $V_A - V_B$ is positive when Q›0

(ii) $V_A - V_B$ is negative when Q‹0

19. What is the work done to move a test charge q through a distance of 1 cm along the equatorial axis of dipole?

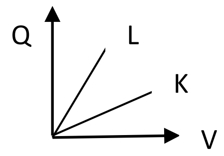
Potential at any point on the equatorial line is 0. Hence work done W = qΔV =0 as ΔV=0

20. **A** 500µC charge is at the centre of square of side 10cm. Find work done in moving a charge of 10 µC between two diagonally opposite points on the square.

Two diagonally opposite points are equidistant from the centre of square hence potential at these points due to given charge will be equal.

W= QΔV=0 as ΔV=0.

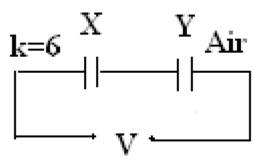
21. The following graph shows variation of charge Q with voltage V for two capacitors K and L. In which capacitor is more energy stored ?



The slope of straight line represents capacitances. Therefore capacity of L will be more.

Energy stored in a capacitor$U = \frac{1}{2}CV^2$

At a given potential energy stored in L is more than that stored in K.



22. In the given figure X, Y represent parallel plate capacitors having the same area of plates and the same distance of separation between them, What is the relation between the energies stored in the capacitors?

$\boldsymbol{C_x = 6C_y}$

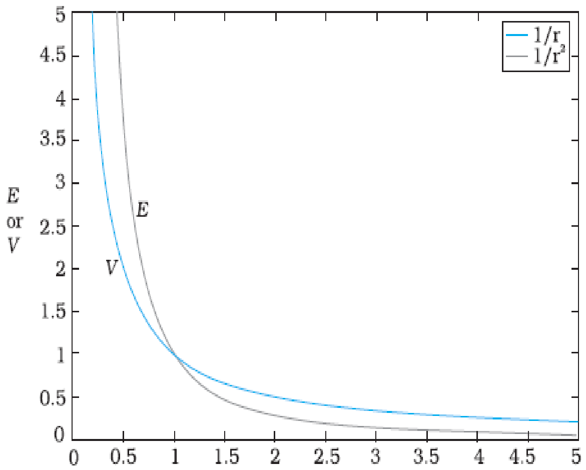
In series combination charge on both the capacitors is same

$U = \frac{Q^2}{2C}$ **Hence** $\frac{U_x}{U_y} = \frac{C_y}{C_x} = \frac{1}{6}$

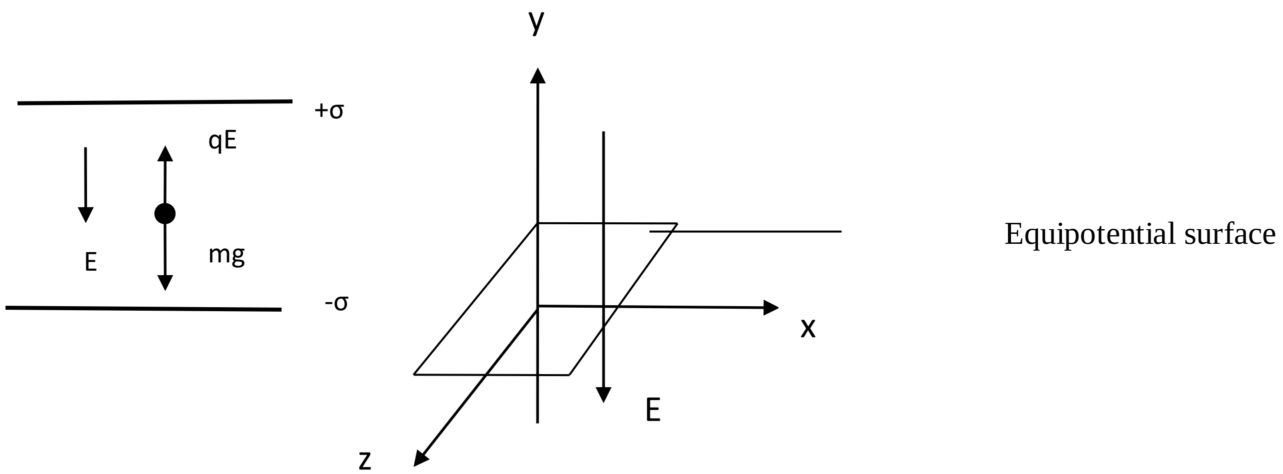
PREVIOUS YEAR QUESTIONS (2 marks)

1. Draw a plot showing variation of
   a) Electric field E and

b) Electric potential V with distance r due to a point charge Q.



2. Two uniformly large parallel thin plates having density +σ and –σ are kept in X Y plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m is and charge –q remains stationary between the plates, What is the magnitude and direction of this field?



Negative charge experiences a force in a direction opposite to electric field

$$qE = mg \ \therefore E = \frac{mg}{q} \ \textit{along vertically downward direction}$$

3. Two charged conducting spheres of radii r_1 and r_2 are connected to each other by a wire. Find the ratio of electric field at the surface of the two spheres.

The two spheres will have same potential that is $V_1 = V_2$

$$k\frac{q_1}{r_1} = k\frac{q_2}{r_2}$$

$$\frac{q_1}{q_{21}} = \frac{r_1}{r_2}$$

Now ratio of electric field $\frac{E_1}{E_2} = \frac{k\frac{q_1}{r_1^2}}{k\frac{q_2}{2}} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1}$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

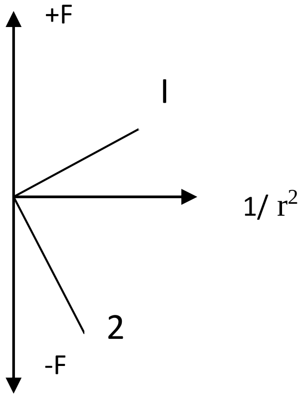
4. Calculate amount of work done in turning an electric dipole of dipole moment $3x10^{-8}$ C-m from position of unstable equilibrium to the position of stable equilibrium in a uniform electric field of intensity 10^3 N/C

For unstable equilibrium $\theta=180^0$ and for stable equilibrium $\theta=0^0$

Required work done

$$W = p\,E\,(\cos\theta_1 - \cos\theta_2)$$
$$W = 3\times10^{-8}\times10^{3}(\cos 180 - \cos 0) = -6\times10^{-5} J$$

5. Plot a graph showing the variation of Coulomb's force (F) versus $1/r^2$ where r is the distance between the two charges of each pair of charge(1µC,2 µC) and (1µC,-3 µC).



For given pair of charge $F \propto \frac{1}{r^2}$

Magnitude of q_1q_2 is higher and negative in second case

6. A spherical conducting shell of inner radius R_1 and outer radius R_2 has a charge Q. A charge q is placed at the centre of the shell.
   i) What is the surface charge density on the a) Inner surface b) Outer surface of the shell.
   ii) Write the expression for the electric field at a point x>R2 from the centre of the shell.
   i) a) Charge produced on inner surface by induction = -q

Surface charge density on inner surface due to induction$\sigma_i = \frac{-q}{4\pi R_1^2}$

An equal amount of charge +q is produced on outer surface

b) Charge on outer surface is =q+Q

Surface charge density on outer surface $\sigma_0 = \frac{q+Q}{4\pi R_2^2}$

ii) Electric field at a distance x (x>R_2) is

$E = k\frac{q+Q}{x^2}$and is directed away from the conductor.

7. A dipole, with its charge,-q and +q, located at the point (o,-b,o) and (o,+b,o), is present in a uniform electric field E. The equipotential surfaces of the field are planes parallel to the Y-Z planes.
   i) What is the direction of the electric field E?
   ii) How much torque would the dipole experience in this field?
   (i) Electric field is along X-asxis as it should be perpendicular to equipotential surface which is in Y-Z plane
   (ii) As dipole is along Y-axis and length of the dipole is 2b

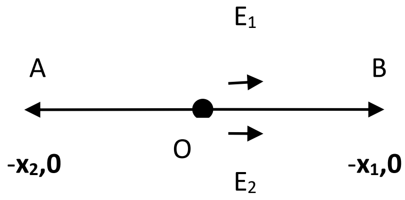
∴ Electric dipole moment is p=q(2b)j

Electric field is given by $\vec{E} = Ei$

Torque $\tau = \vec{p}\times\vec{E} = q(2b)j\times Ei = q2bE(-k)$

Magnitude of torque is $\tau = q2bE$

8. Two charges +Q and –Q are kept at ($-x_2$, o) and (x_1,o) respectively in the X-Y plane. Find the magnitude and direction of the net electric field at the origin(o,o).



+ Q charge is located at A(x_2,0) AND –Q charge is at B(x_1,0)

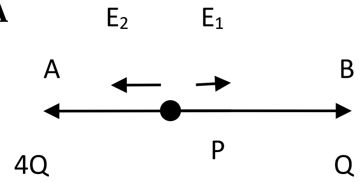
E. F at O due to + Q charge is $E_1 = k\frac{Q}{x_2^2}$ (Towards B)

E. F at O due to - Q charge is $E_2 = k\frac{Q}{x_1^2}$ (Towards B)

Net E. F at O is $E=E_1+E_2$ (Towards B)

$$E = kQ\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]$$

9. Two point charges 4Q, Q are separated by 1m in air. At what point on the line joining the charges is the electric field intensity zero? .



Electric field intensity is zero at P which is at a distance x from A

Electric field intensity E_1 due to 4Q at P = Electric field intensity E_2 due to Q at P

$$k\frac{4Q}{x^2} = k\frac{Q}{(1-x)^2}$$

$$\frac{4Q}{x^2} = \frac{Q}{(1-x)^2} \quad or \quad \frac{2}{x} = \frac{1}{1-x}$$

$2 - 2x = x or\ x = \frac{2}{3}m$ **E F is zero at a distance of $\frac{2}{3}$ m from +4Q charge.**

10. Define electric flux Write its SI unit.

A spherical balloon carries a charge 2μC that is uniformly distributed over its surface. As the balloon is blown and increases in size, how will the electric field coming out of surface change? Give reason.

It is represented by electric field passing normally through a given surface. SI unit of flux is newton m^2/coulomb.

According to Gauss law electric flux through a closed surface is given by $\emptyset = \frac{q}{\epsilon_0}$

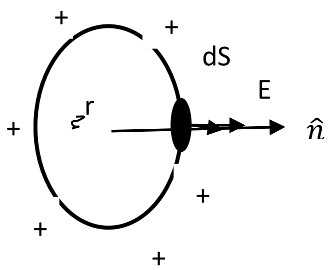
The total flux coming out of the surface remains constant because it depends only on the charge enclosed by the surface.

11. Show that the electric field at the surface of charged conductor is given by $E = \frac{\sigma}{\epsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface of the conductor.

Let q charge be distributed uniformly on a surface of radius r Surface charge density $\sigma = \frac{q}{4\pi r^2}$

Electric field intensity on the surface of shell is $E = \frac{q}{4\pi\epsilon_0 r^2}$

Or $E = \frac{\frac{q}{4\pi r^2}}{\epsilon_0} = \frac{\sigma}{\epsilon_0}$

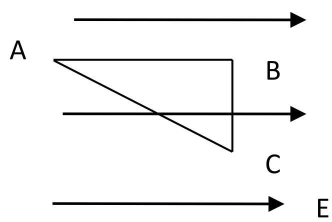


12. A cubical Gaussian surface encloses a charge $8.85\text{x}10^{-10}$C in vacuum. Calculate electric flux through one of its faces.

$$\emptyset = \frac{q}{\epsilon_0} = \frac{8.85\text{x}10^{-10}}{8.85\times10^{-12}} = 100Nm^2/C$$

Flux through each face of cube is=$\frac{\emptyset}{6} = \frac{100}{6} = 16.7\ Nm^2/C$

13. A test charge q is moved without acceleration from A to C along the path from A to B and then to C in electric field E as shown in the figure.
(i) Calculate potential difference between A and C.
(ii) At which point is the electric potential more and why?



(i) AB=4cm BC=3cm AC=5cm

$\Delta V = -E\Delta r = -(2+2)E$

V_A -V_C=-4E

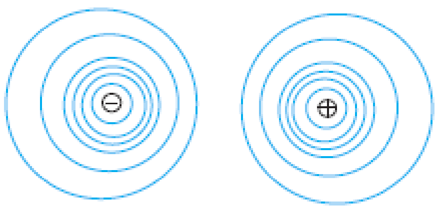
(ii) As potential decreases in the direction of electric field hence potential at C is greater than that at A

14. Two point charges +3µC and -3µC are placed 5cm apart

(i) Draw surface of the system.

(ii) Why do equipotential surface get closer near the point charge?

(i)

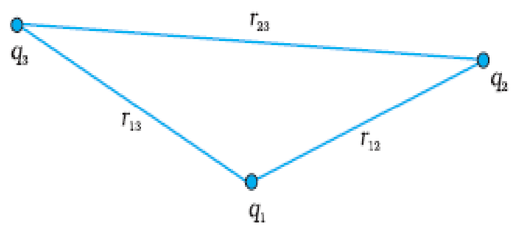


(iii) Electric field near the charge is stronger hence the surfaces get closer

$$E = -\frac{dV}{dr}$$

$$E \propto \frac{1}{dr}$$

15. Find out the expression for the potential energy of a system of three charges q_1,q_2 and q_3 located at r_1,r_2 and r_3 with respect to common origin O .



Potential energy of the charges q_1 and q_2

$$U_{12} = k\frac{q_1q_2}{r_{12}}$$

Similarly

$$U_{23} = k\frac{q_2q_3}{r_{23}}$$

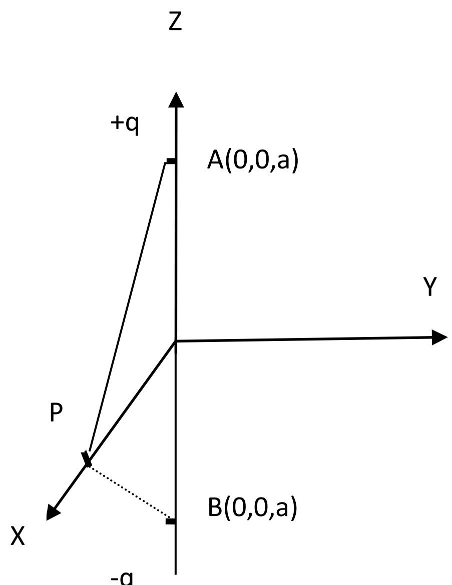
$$U_{13} = k\frac{q_1q_3}{r_{13}}$$

∴ Net potential energy of the system

$$U_{123} = U_{12} + U_{23} + U_{13}$$

$$U_{123} = k\frac{q_1q_2}{r_{12}} + k\frac{q_2q_3}{r_{23}} + k\frac{q_1q_3}{r_{13}}$$

16. Can two equipotential surfaces intersect each other? Give reasons. Two charges –q and +q are located at points A (0,0,-a) and B(0,0,+a) respectively. How much work is done in moving a test charge from point P(7,0,0) to Q(-3,0,0)?



No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on the two surfaces which gives two directions of E at the same point which is not possible.

Every point on X- axis is on the equatorial line of dipole hence potential at every point is zero.

$W = q\Delta V = q \times 0 = 0$

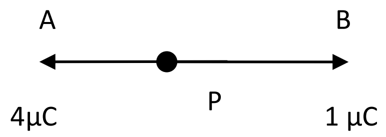
17. Two point charges 4μC and -2 μC are separated by a distance of 1m. At what point on the line joining the two charges is the electric potential zero.

Let the potential be zero at a point P at a distance x from the charge 4μC.

At P $V_1+V_2=0$

$$k\frac{q_1}{r_1}+k\frac{q_2}{r_2}=0$$

$$k\frac{4\times10^{-6}}{x}-k\frac{1\times10^{-6}}{1-x}=0$$



$$\frac{4}{x}=\frac{1}{1-x}\ or\ 2(1-x)=\ x$$

$$2=3x\ or\ x=\frac{3}{2}m$$

Potential is zero at a distance of $\frac{3}{2}m$ from 4μC charge

18. Net capacitance of three identical capacitors in series is 1μF. What will be their net capacitance if connected in parallel? Find the ratio of energy stored in the two configurations if they are both connected to the same source.

If n capacitors each of capacity C are connected in series then the equivalent capacity is given by

$C_s=\frac{C}{3}$ **or** $C=3\times1=3$μF

And when connected in parallel

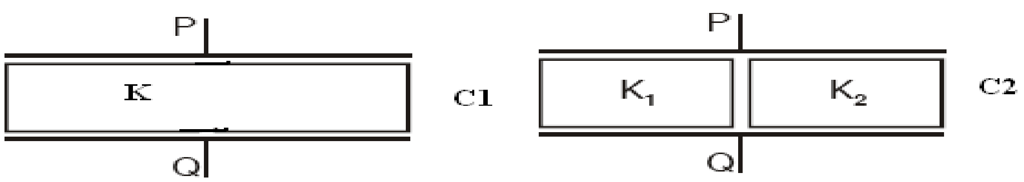
$C_p=3C=3\times3=9$

V remaining constant energy stored in capacitor is $U=\frac{1}{2}CV^2$

Ratio of capacity in series and parallel combination

$$\frac{U_s}{U_p}=\frac{C_S}{C_p}=\frac{1}{9}$$

19. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitance C each. The space between their plates is now filled with dielectric as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constant K, K_1 and K_2 .



After inserting dielectric let their capacitance be C_1' and C_2'

$C_1'=kC$ and $C_2'=\frac{k_1\in_0A/2}{d}+\frac{k_2\in_0A/2}{d}=\frac{\in_0A}{d}\left(\frac{k_1}{2}+\frac{k_2}{2}\right)$

C2 acts as if two capacitors each of area A/2 and separation d are connected in parallel

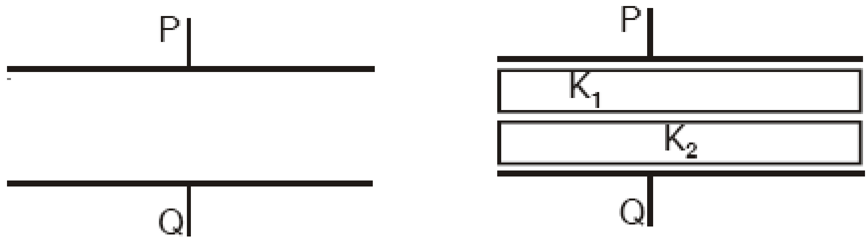
$$C_2'=C\left(\frac{k_1}{2}+\frac{k_2}{2}\right)$$

Given $C_1'=C_2'$

Hence $kC=\ C\left(\frac{k_1}{2}+\frac{k_2}{2}\right)\ or$

$$k = \left(\frac{k_1}{2} + \frac{k_2}{2}\right)$$

20. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with the slabs of dielectric constant K_1 and K_2 as shown in C_2. Find the capacitance of the capacitor C_2 if area of the plates is A, distance between the plates is d.



After introduction of dielectric capacitor acts as if two capacitors each of area and separation d/2 are connected in series

$$C_1 = \frac{\epsilon_0 A}{d}$$

$$\frac{1}{C_2} = \frac{1}{\left(k_1 \frac{\epsilon_0 A}{d/2}\right)} + \frac{1}{\left(k_2 \frac{\epsilon_0 A}{d/2}\right)}$$

$$\frac{1}{C_2} = \frac{d}{\epsilon_0 A}\left(\frac{1}{2k_1} + \frac{1}{2k_2}\right)$$

$$\frac{1}{C_2} = \frac{1}{2C_1}\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$C_2 = C_1\left(\frac{2k_1 k_2}{k_1 + k_2}\right)$$

21. Two capacitors of capacitance of 6μF and 12μF are connected in series with a battery. The voltage across the 6μF capacitor is 2V. Compute the total battery voltage.

C_1=6μF. V_1=2V

$q_1 = C_1 V_1$=6μFx2V=12μC

Charge on 12μF capacitor q_2 =12μC

Hence $V_2 = \frac{12\mu C}{12\mu F} = 1V$

Net potential difference

$V = V_1 + V_2$= 3+1=3V

22. A parallel plat capacitor with air between the plate has a capacitance of 8pF. The separation between the plates is now reduced by half and the space between them is filled with the medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.

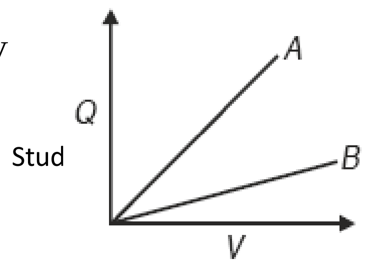
$C = \frac{\epsilon_0 A}{d} = 8pF$ **Where A is area of plates and d is the distance between them**

$$Now\ C' = k\frac{\epsilon_0 A'}{d'} = 5\frac{\epsilon_0 A}{\frac{d}{2}}$$

$$C' = 10\frac{\epsilon_0 A}{d} = 10C$$

$$C' = 10 \times 8 = 80pF$$

23. The given graph shows the variation of charge, q versus potential difference V for capacitors C_1 and C_2 . The two capacitors have same plate area of C_2 is



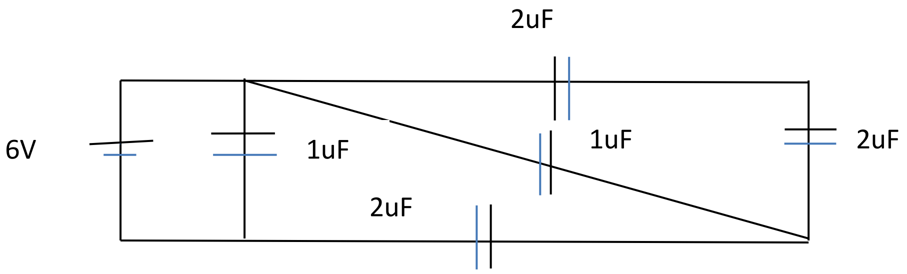
double than that C_1. Which of the lines in the graph correspond to C_1 and C_2 and why?

$$C = \frac{\epsilon_0 A}{d}$$

$$C_2 = 2C_1$$

The slope of graph represents capacity of capacitor
A has greater slope than that of B
So capacitance of A is greater than that of B

24. Find the total capacitance of the capacitors in the given network.



The equivalent capacitance of C_1 and C_2 (C') = $C_1 C_2 / (C_1 + C_2)$

C'=$\frac{2\times2}{2+2} = 1\mu F$

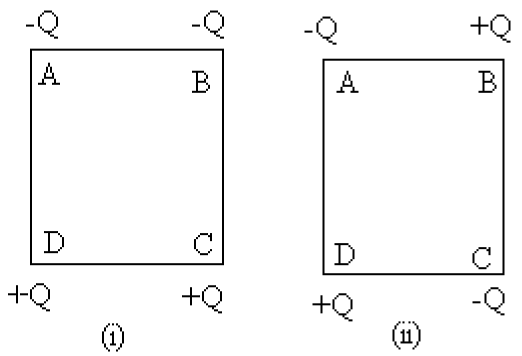
C' is parallel with C_3 ,so the equivalent capacitance of C_1 , C_2 and C_3 is C"=1+1=2μF.

C" is in series with C_4, thus the equ. capacitance (C"')= C_4C"/C_4 + C" = 1μF.

This in parallel with C_5 so eq. capacitance across AB is ,C_{AB}, =1+1=2μF.

HOTS ON ELECTROSTATICS

1. Four point charges are placed at the four corners of a square in the two ways (i) and (ii) as shown below. Will the (i) electric field (ii) Electric potential, at the centre of the square, be the same or different in the two configrations and why? (2)



(I) **Electric field is a vector quantity . in the first case electric field at the center due to charges at A and due to C add up and also due to charges at B and D are added up. There exists electric field at the centre. Where as in the second case field due to A and C are equal and opposite and also due to B and D are also equal and opposite, so the resultant field is zero.**

(II) **Potential is a sclar quantity and is positive due to positive charge and negative due to negative charge. Hence resultant potential at the centre is zero in both the cases.**

2. Find the ratio of potential difference that must be applied across the parallel and series combination of two capacitors C1 and C2 with their capacitance in the ratio 1:3 so that the energy stored in the two cases is same.

Given $\frac{C1}{C2} = \frac{1}{3}$

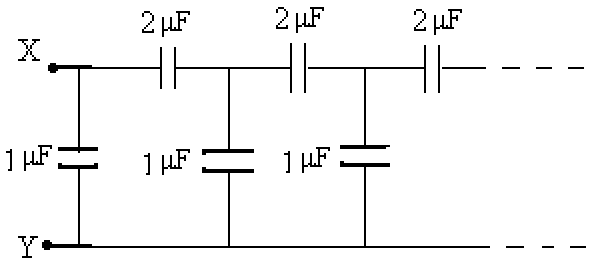
$U_P = U_S$

$\frac{1}{2} C_P V_P^2 = \frac{1}{2} C_S V_S^2$

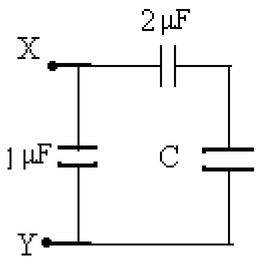
$\frac{V_P^2}{V_S^2} = \frac{C_S}{C_P}$ **Substituting** $C_S = \frac{C_1 C_2}{C_1 + C_2}$ **And** $C_p = C_1 + C_2$

$\frac{V_P}{V_S} = \frac{\sqrt{3}}{4}$

3. Find the capacitance of the infinite ladder between the points X and Y



Let the equivalent capacitance of the network be C Since it is an infinite network addition one such more unit will not affect equivalent capacitance. The network will appear like



The equivalent capacity of the new arrangement must be C.

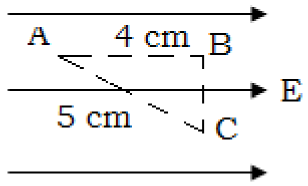
$$C = 1 + \frac{2 \times C}{2 + C}$$

Or C^2- C- 2=0

Therefore C=2 µF or -1µF

As capacitance cannot be negative therefore C=2µF

4. Three points A, B and C lie in uniform electric field E of $5x10^3$N/C as shown in the figure. Find potential difference between A and C.



Points B and C lie on equipotential surface. So $V_C = V_B$

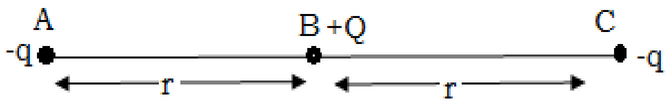
Potential difference between A and C = Potential difference between A and B

$$= -E\Delta x$$

$$= -5 \times 10^3 \times 4 \times 10^{-2} = -200V$$

5. Three charges –q, +Q and –q are placed at equal distance on a straight line. If the potential energy of the system of three charges is zero, find the ratio of Q:q

As total potential energy is zero

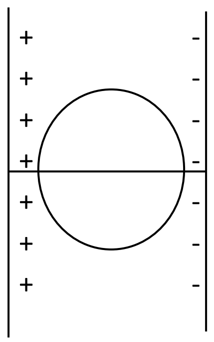


$$\frac{1}{4\pi\epsilon_0}\left[\frac{-qQ}{r} + \frac{(-q)(-q)}{r} + \frac{Q(-q)}{r}\right] = 0$$

$$-Q + \frac{q}{2} - Q = 0$$

$$\left(\frac{Q}{q} = \frac{1}{4}\right)$$

6. .A metallic sphere placed between two charged metallic plates. A student draws the lines of force as shown in figure. Is he correct?



No, because within a metal the electric field is zero. Therefore, no lines of force should exist between the spheres.

7. Two charged spherical conductors, each of radii R, are distance d apart such that d is slightly greater than 2R. They carry charge +q and -q. Will the force of attraction between them be exactly $\frac{q^2}{4\pi\epsilon_0 d^2}$?

No, the force of attraction between spherical conductors will be more than $K.q^2/d^2$, due to attractive of opposite charges; these will be redistribution of charges on spheres. Obviously, the effective distance between the charges will be reduced and hence effective force will be increased.